الامتحان الوطني الموحد للبكالوريا
المسالك الدولية – خيار أنجليزية
الدورة العادية 2019
- الموضوع -
NS22E

المملكة المغربية
وزارة التربية الوطنية والتكوين المهني والتعليم العالي والبحث العلمي
المركز الوطني للتقويم والامتحانات والتوجيه

المادة	الرياضيات	مدة الانجاز	3
الشعبة أو المسلك	مسلك علوم الحياة والأرض ومسلك العلوم الفيزيائية – خيار أنجليزية	المعامل	7

GENERAL INSTRUCTIONS

- ✓ The use of non- programmable calculator is allowed ;
- ✓ The exercises can be treated in the preferred order by the candidate ;
- ✓ The use of red color when writing solutions is to be avoided.

COMPONENTS OF THE EXAM

- ✓ The exam consists of three exercises and a problem , independent of each other according to the fields as follows :

Exercise 1	Geometry in space	3 points
Exercise 2	Complex numbers	3 points
Exercise 3	Calculating probabilities	3 points
Problem	Study of numerical function, calculating integrals and numerical sequences	11 points

- ✓ ln denotes the Napierian logarithm function

Exercise 1 : (3 points)

In the space referred to an orthonormal direct coordinate system $\left(O,\vec{i},\vec{j},\vec{k}\right)$, we consider the points $A(1,-1,-1)$, $B(0,-2,1)$ and $C(1,-2,0)$

0.75 1)a) Show that $\overrightarrow{AB}\wedge\overrightarrow{AC}=\vec{i}+\vec{j}+\vec{k}$

0.5 b) Deduce that $x+y+z+1=0$ is a cartesian equation of the plane (ABC)

0.75 2) Let (S) the sphere of equation $x^2+y^2+z^2-4x+2y-2z+1=0$

Show that the center of the sphere (S) is the point $\Omega(2,-1,1)$ and that its radius is $R=\sqrt{5}$

0.5 3)a) Calculate $d(\Omega,(ABC))$ the distance of the point Ω to the plane (ABC)

0.5 b) Deduce that the plane (ABC) intersects the sphere (S) along a circle (Γ)

(the determination of the center and radius of the circle (Γ) is not required)

Exercise 2 : (3 points)

0,75 1) Solve in the set of complex numbers $\mathbb{C}$ the equation : $z^2-2z+4=0$

2) In the complex plane referred to an orthonormal direct coordinate system $(O,\vec{u},\vec{v})$,

we consider the points A , B, C and D of respective affixes $a=1-i\sqrt{3}$, $b=2+2i$, $c=\sqrt{3}+i$ and $d=-2+2\sqrt{3}$

0,5 a) Verify that $a-d=-\sqrt{3}(c-d)$

0,25 b) Deduce that the points A , C and D are collinear

0,5 3) Let z be the affix of a point M in the complex plane and z' the affix of the point M' image of M by the rotation R with center O and angle $-\dfrac{\pi}{3}$

Verify that $z'=\dfrac{1}{2}az$

4) Let the point H the image of the point B by the rotation R , and h its affix, and P the point of affix p such that $p=a-c$

0,5 a) Verify that $h=ip$

0,5 b) Show that the triangle OHP is rectangle and isosceles in O .

Exercise 3 : (3 points)

An urn contains ten balls : three green balls, six red balls and one black ball. All the balls are indistinguishable to the touch .

We draw randomly and simultaneously three balls from the urn.

	We consider the following events: A : " Get three green balls " B : " Get three balls of the same color" C : "Get at least two balls of the same color "
2	1) Show that $p(A)=\dfrac{1}{120}$ and $p(B)=\dfrac{7}{40}$
1	2) Calculate $p(C)$

Problem : (11 points)

First part

We consider the numerical function f defined on $]0,+\infty[$ by $f(x)=x+\frac{1}{2}-\ln x+\frac{1}{2}(\ln x)^2$

and (C) the curve of f in an orthonormal coordinate system $(O,\vec{i},\vec{j})$ (unit: $1cm$)

0.5 1) Calculate $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ and then interpret geometrically the obtained result

0.25 2)a) Verify that for every x on $]0,+\infty[$, $f(x)=x+\frac{1}{2}+\left(\frac{1}{2}\ln x-1\right)\ln x$

0.5 b) Deduce that $\lim\limits_{x\to+\infty} f(x)=+\infty$

0.5 c) Show that for every x on $]0,+\infty[$, $\dfrac{(\ln x)^2}{x}=4\left(\dfrac{\ln\sqrt{x}}{\sqrt{x}}\right)^2$

and then deduce that $\lim\limits_{x\to+\infty}\dfrac{(\ln x)^2}{x}=0$

0.75 d)Show that the curve (C) admits a parabolic branch in the direction of the line (Δ) wich an equation is $y=x$ at $+\infty$

0.5 3)a) Show that for every x on $]0,1]$: $(x-1)+\ln x\le 0$

and for every x on $[1,+\infty[$: $(x-1)+\ln x\ge 0$

1 b)Show that for every x on $]0,+\infty[$, $f'(x)=\dfrac{x-1+\ln x}{x}$

0.5 c) Set up the table of variations of the function f

0.5 4)a)Show that $f''(x)=\dfrac{2-\ln x}{x^2}$ for every x on $]0,+\infty[$

0.5 b) Deduce that the curve (C) admits an inflection point wich coordinates will be determined

0.5	5) a) Show that for every x on $]0,+\infty[$, $f(x)-x=\frac{1}{2}(\ln x-1)^2$,
	and then deduce the relative position of the line (Δ) and the curve (C)
1	b) Sketch the line (Δ) and the curve (C) in the same system coordinate $(O,\vec{i},\vec{j})$
0.5	6) a) Show that the function $H: x \mapsto x\ln x - x$ is a primitive of the function $h: x \mapsto \ln x$ on $]0,+\infty[$
0.75	b) Using an integration by parts, show that $\int_1^e (\ln x)^2 dx = e-2$
0.5	c) Calculate, in cm^2, the area enclosed between the curve (C) , the line (Δ), and the lines of equations $x=1$ and $x=e$
	Second part :
	Let (u_n) be the numerical sequence defined by $u_0=1$ and $u_{n+1}=f(u_n)$ for every natural number n
0.5	1) a) Show by induction that $1 \le u_n \le e$ for every natural number n
0.5	b) Show that the sequence (u_n) is increasing
0.5	c) Deduce that the sequence (u_n) is convergent.
0.75	2) Calculate the limit of the numerical sequence (u_n).